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Langevin Transducer Analysis and Acoustic Levitation

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Langevin Transducer Analysis and Acoustic Levitation

Abstract
In this experiment we investigated the behavior of a 40W commercial, ultrasonic transducer. We observed a frequency response hysteresis near the resonant frequency of 28 kHz and determined that the resonant frequency depends on temperature and driving amplitude. We then used the transducer to create a resonant standing wave to levitate small objects including water drops. We show the non-linear acoustical theory to support the acoustic levitation force being produced by a nonzero time-average pressure. Using Schlieren optics we were able to observe pressure variations and measured relative pressures using a microphone. From the dynamical behavior of a levitated object, we were able to estimate the pressure amplitude of the standing wave.

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LAKE FOREST COLLEGE

Senior Thesis

Langevin Transducer Analysis and Acoustic Levitation

by

Robert Dean Mecham

April 25, 2018

The report of the investigation undertaken as a Senior Thesis, to carry two courses of credit in the Department of Physics

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ABSTRACT

In this experiment we investigated the behavior of a 40W commercial, ultrasonic transducer. We observed a frequency response hysteresis near the resonant frequency of 28 kHz and determined that the resonant frequency depends on temperature and driving amplitude. We then used the transducer to create a resonant standing wave to levitate small objects including water drops. We show the non-linear acoustical theory to support the acoustic levitation force being produced by a nonzero time-average pressure. Using Schlieren optics we were able to observe pressure variations and measured relative pressures using a microphone. From the dynamical behavior of a levitated object, we were able to estimate the pressure amplitude of the standing wave.
DEDICATION

I would like to dedicate this work to several of my family members. To my grandparents, thank you for all the sacrifices you made to take me in and provide me with the opportunity to attend college. To my mother and stepfather, you guys were the reason I was able to afford going to school. You both gave a large portion of your income to give me this chance and I cannot thank you enough. Lastly, I would like to thank my father for always helping me financially when I was in a mind. This thesis is a representation of my hard work and it is a sign of my appreciation for the opportunity you all gave me.
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I. INTRODUCTION

In linear acoustics the time-average excess pressure of a pressure wave is zero; this is because at a given point the pressure is fluctuating above and below the ambient pressure in equal amounts. However, near pressure nodes of a standing wave the linear terms in the pressure wave go to zero and the nonlinear effects prevail. In nonlinear acoustics the time-average of the excess pressure is nonzero. The acoustic levitation force is a result of nonlinear acoustic pressure waves on a small object. Acoustic levitation provides easy access to microgravity environments and allows for the containerless processing\(^1\) of various materials.

In this experiment we use an ultrasonic Langevin transducer to establish a standing pressure wave and we are able to levitate small objects just below the pressure nodes. To maintain a stable microgravity environment, we need to understand the behavior of our Langevin transducers and eliminate symmetries that diminish the lateral restoring forces on a levitated object. Stable levitation will allow for consistent trials in examining the properties and capabilities of the acoustic levitation force.
II. THEORY OF ACOUSTIC LEVITATION

The Eulerian pressure refers to the pressure at a fixed point in space and we will denote this as $P$. If $P_0$ represents the ambient air pressure, then we can represent the time-average *mean excess pressure* at a given position as $\langle P - P_0 \rangle$. For nonlinear effects the excess pressure will be non-zero and depend on $\theta$ and as a result, a force will be exerted on an object with surface area $S$ in the presence of this pressure. This exerted force on the object is the acoustic levitation force and it points towards regions of lower pressure. If we take this object to be spherically symmetric then there is no $\phi$ dependence and the $x$ and $y$ components of the force will cancel so, the $z$-component of the force will be

$$F_z = -\int \langle P - P_0 \rangle \cos \theta dS$$

(1)

where $\theta$ is the polar angle to the $z$-axis. Following the presentation in chapter 6 of *Nonlinear Acoustics*, we will determine an expression for the mean excess pressure when a small, spherical object is in a standing pressure wave.

Consider the isentropic expansion for an ideal fluid (air)

$$\frac{1}{\rho'} P = \frac{1}{\rho_0'} P_0$$

(2)

where $\rho = \rho_0 + \rho'$ is the sum of ambient density and excess density respectively,

$P = P_0 + p$ is the sum of the ambient and acoustic pressures respectively and $\gamma$ is the ratio of specific heats. Rearranging Eq.(2) we find

$$P - P_0 = P_0 \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right].$$

(3)
The speed of sound in an isentropic fluid can be expressed as

\[
c^2 = \left( \frac{\partial P}{\partial \rho} \right)_{s, \text{const}} = \frac{\partial}{\partial \rho} \left[ P_0 \left( \frac{\rho}{\rho_0} \right)^\gamma \right] = P_0 \gamma \frac{1}{\rho} \left( \frac{\rho}{\rho_0} \right)^{\gamma-1}
\]  \tag{4}

and at equilibrium

\[
c_0^2 = P_0 \gamma \frac{1}{\rho_0} \rightarrow P_0 = \frac{\rho_0 c_0^2}{\gamma}
\]

For an ideal fluid we have two equations of motion: the momentum equation and the continuity equation. The momentum equation is an analog to Newton’s second law when considering a force per volume, and it is written as

\[
\rho \left[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial P}{\partial x_i},
\]  \tag{5}

where there is an implicit sum over \( j \), the first term is similar to \( ma \) and the second term is nonlinear. The continuity equation is

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0.
\]  \tag{6}

In Eqs.(5) and (6) \( u_i \) is the \( i \)th component of the particle velocity vector, where a “particle” is a collection of air molecules, and \( x_i \) is a component of the position vector.

The sound field in an inviscid fluid is irrotational, which means that we may express our particle velocity as the gradient of some velocity potential, namely \( \mathbf{u} = \nabla \phi \). Using the velocity potential we may rewrite Eq.(5) as

\[
\nabla \left[ \frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla \phi|^2 \right] = -\frac{\nabla P}{\rho}
\]  \tag{7}

where \( \frac{1}{2} |\nabla \phi|^2 \) is the nonlinear term from Eq.(5).
For an adiabatic process the change in entropy, $ds$, is zero and the enthalpy per unit mass, $w$ can be written in terms of the pressure,

$$dw = Tds + \frac{dP}{\rho} \rightarrow \tilde{\nabla}w = \frac{\nabla P}{\rho}. \tag{8}$$

Combining Eq. (7) and (8) we get

$$w = -\frac{\partial \phi}{\partial t} - \frac{1}{2} \left| \nabla \phi \right|^2 + C' \tag{9}$$

where $C'$ is independent of position and may be needed to satisfy a constraint on a system; however, in our case the system is open so there is no need for a nonzero value.

We then expand $P$ in a Taylor series in $w$:

$$P = P_0 + \left( \frac{\partial P}{\partial w} \right)_{s,0} w + \frac{1}{2} \left( \frac{\partial^2 P}{\partial w^2} \right)_{s,0} w^2 + \cdots. \tag{10}$$

The subscript $s,0$ indicates that the term is evaluated at constant entropy and at equilibrium. From Eq. (8) we can see that $\left( \partial P/\partial w \right)_s = \rho$ and from (4) and (8) we can see that $\left( \partial^2 P/\partial w^2 \right)_s = \left( \partial \rho/\partial P \right)_s \left( \partial P/\partial w \right)_s = \rho/c^2$. Substituting these values into Eq. (10) and using $C' = 0$ we get

$$P = P_0 + \rho_0 \left( -\frac{\partial \phi}{\partial t} - \frac{1}{2} \left| \nabla \phi \right|^2 \right) + \frac{1}{2} \rho_0 \frac{c_0}{2c^2} \left( -\frac{\partial \phi}{\partial t} - \frac{1}{2} \left| \nabla \phi \right|^2 \right)^2 + \cdots. \tag{11}$$

In linear acoustics, $\left| \nabla \phi \right|^2 = 0$ so Eq. (9) becomes $w = P/P_0 = -\partial \phi/\partial t$ and, ignoring the quadratic term in Eq. (11), the average excess pressure is zero. However, in nonlinear acoustics we cannot have $\left| \nabla \phi \right|^2 = 0$ and if we take the time-average of Eq. (11) and only keep the second order terms we see

$$\langle P - P_0 \rangle = \frac{1}{2} \frac{\rho_0}{c_0^2} \left( \left\langle \left( \frac{\partial \phi}{\partial t} \right)^2 \right\rangle \right) - \frac{1}{2} \rho_0 \left\langle \left| \nabla \phi \right|^2 \right\rangle. \tag{12}$$
At second order we can replace our quadratic $\phi$ terms with $\bar{u} = \nabla \phi$ and $\partial \phi / \partial t = -p / \rho_0$.

The mean excess pressure is then defined to be

$$
\langle P - P_0 \rangle = \frac{1}{2\rho_0 c_0^2} \langle p^2 \rangle - \frac{\rho_0}{2} \langle \bar{u} \cdot \bar{u} \rangle
$$

(13)

where $\rho_0$ is the ambient air density, $c_0$ is the speed of sound in air, $p$ is the acoustic pressure and $\bar{u}$ is the air particle velocity.

The acoustic pressure can be expressed as

$$
p = p_0 e^{-i\omega t}
$$

(14)

where

$$
p_0 = p_{10} + p_{r0}.
$$

(15)

The incident and scattered pressure waves are to $p_{10}$ and $p_{r0}$ respectively. To evaluate this expression we need to find the acoustic pressure and the air particle velocity when our spherical object is in the standing pressure wave.
A. Rigid Sphere in a Standing Wave

Consider a rigid sphere of radius \( R \) where \( kR \ll 1 \) and \( k \) is the wavenumber \( \left( \frac{2\pi}{\lambda} \right) \) of the soundwave. Let the incident pressure of the soundwave be \( p_i = p_{i0}e^{-i\omega t} \), where

\[
p_{i0} = A \sin k\zeta \tag{16}
\]

and \( A \) is the incident pressure amplitude. With this we can see that at \( \zeta = 0 \) the pressure is zero, giving the location of a pressure node. Consider a coordinate system in which the positive \( \zeta \) axis points upward, let \( \theta = 0 \) be in the positive \( \zeta \) direction and let \( r = 0 \) at the center of the sphere. If we take the center of the sphere to be at \( \zeta = Z \) then for a point \( (r, \theta), \zeta = Z + r\cos\theta \). Substituting this value of \( \zeta \) into Eq.(16) we find

\[
p_{i0} = \frac{A}{2i} \left[ e^{ik(Z+r\cos\theta)} - e^{-ik(Z+r\cos\theta)} \right]. \tag{17}
\]

Using the identity \( 3 \)

\[
e^{ikr\cos\theta} = \sum_{n=0}^{\infty} (2n+1)t^n j_n(kr)P_n(\cos\theta) , \tag{18}
\]

we can then express Eq.(17) as

\[
p_{i0} = \sum_{n=0}^{\infty} (2n+1)A_n j_n(kr)P_n(\cos\theta) , \tag{19}
\]

where
with \( j_n \) and \( P_n \) being the spherical Bessel function and Legendre polynomial respectively.

The wave that scatters from the surface of the sphere can be described as

\[
p_r = p_{r0} e^{-i\omega t}, \text{ where } p_{r0} \text{ can be expressed as}
\]

\[
p_{r0} = \sum_{n=0}^{\infty} B_n j_n^{(1)}(kr) P_n(\cos \theta).
\]

Here \( j_n^{(1)} \) is the spherical Hankel function of the first kind and together with the time dependence, \( e^{-i\omega t} \), describes a spherical wave propagating outward from the center. At the surface of the sphere the total acoustic pressure, \( p = (p_{r0} + p_{i0}) e^{-i\omega t} \) has a zero normal derivative; therefore, the normal component for the velocity of the air at the surface will also be zero. The normal derivative of the pressure imposes the boundary condition

\[
\left( \frac{\partial p_{r0}}{\partial r} \right)_{r=R} = -\left( \frac{\partial p_{i0}}{\partial r} \right)_{r=R}.
\]

If we apply this boundary condition to Eqs. (19) and (21) then for terms with the same \( n \) we find

\[
B_n = -\frac{(2n+1) j_n'(kR)}{j_n^{(1)'}(kR)} A_n.
\]

Using Eqs. (19), (21) and (23) we can determine the sound pressure at the surface of the sphere

\[
p_0 = \sum_{n=0}^{\infty} (2n+1) \left[ j_n(kR) - \frac{j_n'(kR) h_n^{(1)}(kR)}{h_n^{(1)'}(kR)} \right] A_n P_n(\cos \theta).
\]

For small \( kR \) and keeping terms up to order \((kR)^2\), Eq.(24) becomes
\[
 p_0 = \left[1 - \frac{(kR)^2}{2}\right] \sin kZ + \frac{3}{2} (kR) A \cos(kZ) P_1(\cos \theta) - \frac{5}{9} (kR)^2 A \sin(kZ) P_2(\cos \theta) .
\] (25)

At \(r=R\) the tangential component of the particle velocity becomes

\[
 u_\theta = u_{\theta 0} e^{-i\omega \theta}
\] (26)

and using Newton’s Second Law one can show that

\[
 u_{\theta 0} = \frac{1}{i\omega \rho_0 R} \frac{\partial p_0}{\partial \theta} .
\] (27)

Substituting Eq.(25) into Eq. (27) we find

\[
 u_{\theta 0} = \frac{3i}{2} \frac{A}{\rho_0 c_0} \cos kZ \sin \theta - \frac{5i}{3} \frac{(kR)A}{\rho_0 c_0} \sin kZ \cos \theta \sin \theta .
\] (28)

Equations (25) and (28) can be substituted into Eq. (13) to find the mean excess pressure at \(r=R\):

\[
 \langle P - P_0 \rangle = \frac{A^2}{4 \rho_0 c_0^2} \left[ \sin^2 kZ + \frac{3}{2} (kR) \sin 2kZ \cos \theta - \frac{9}{4} \cos^2 kZ \sin^2 \theta + \frac{5}{2} (kR) \sin 2kZ \sin^2 \theta \cos \theta \right] .
\] (29)

Using \(dS = 2\pi R^2 \sin \theta d\theta\) we can now find the force on the spherical particle by substituting Eq.(29) into Eq.(1) and integrating over \(\theta\). Thus, the \(z\) component of the acoustic levitation force on our sphere is

\[
 F_z = -\frac{5\pi}{6} \frac{A^2 kR^3}{\rho_0 c_0^2} \sin 2kZ .
\] (30)

This force is consistent with that of a restoring force where the object is in equilibrium at \(Z = 0\), which is a pressure node in this case.
At the core of the acoustic levitation system lies the piezoelectric transducer. This transducer works by having a voltage applied across a piezoelectric material. If a voltage is applied across the crystal, the dipoles will align themselves with the electric field and the crystal expands. When pressure is applied to the crystal the electric dipoles within it re-orient, creating a voltage (FIG. 3).

These piezo crystals are an effective way of converting electrical energy into mechanical energy and will serve as the driver for the transducer’s motion. A steel bolt holds the transducer together and also completes the circuit between the two piezos. In this experiment we used a sinusoidal current to drive the piezo. The transducers we are using are called Langevin transducers and they behave like a spring with free, equal masses on each end. As is apparent in Fig.
2, the masses do not need to have the same shape or density. In our case, the upper frustum of the transducer is an alloy of aluminum and the lower cylinder is made of steel and both have a mass of about 0.24 kg. The primary constraint on the shape is that the height of the frustum must be an integer multiple of half of the resonant wavelength in aluminum. The shape of the upper mass effects the transmission of the wave between the transducer and its surroundings. For air the shape that couples most effectively is an exponential horn similar to that of the bell on a brass instrument. Unfortunately, we did not have the means to produce such a transducer over the course of our experiment. Innovations to the transducer model may be made in future work.
B. Single Transducer System

The bulk of this project was conducted in the single transducer arrangement. Here the transducer was placed an integer number of half wavelengths, in air, from the reflector. We were able to fine tune the distance between the two by mounting the transducer on a ThorLabs precision translation stage. When the incident wave from the transducer reflects off of the curved reflector the soundwave is phase shifted by 180° and creates a standing wave between the two surfaces. From Eq. (30) we know that the acoustic levitation force causes objects to be levitated at the pressure nodes of this standing wave. The curved reflector helps to eliminate planar symmetry of the standing wave; planar symmetry yields a more unstable trapping force as the object is free to move anywhere in the nodal plane. The curved reflector that we used was a glass petri dish, with a radius of curvature of 14.66 cm, which had been mounted to an idle transducer. At the later stages of the experiment we milled one of the transducers to give the surface a radius of curvature of 13.21 cm. We were then able to drive the curved transducer while still reducing planar symmetry and use a plane reflector to establish a standing wave. It is important to note that once the transducer was milled, its mass and height changed leading to a change in its resonant frequency. The resonant frequency of the curved transducer is \( \approx 28.5 \) kHz. This is higher...
than the unmodified transducer resonance of ≈28.0 kHz but it is consistent with the fact that mechanical systems have a resonant frequency that is inversely proportional to the square root of the mass, \( f \propto \frac{1}{\sqrt{m}} \).

To drive the transducer, we used an Agilent 33522A Waveform Generator (and a series of amplifiers) with a signal amplitude of 2.5-3.0 Vpp. To achieve levitation the transducer needed a much larger voltage amplitude, so we used our signal from the 33522A as the input for an AA Lab A-301 HV Amplifier. A gain of 20 was provided by the A-301 and this yielded signal amplitudes of 50-60 Vpp; however, it can only source 100 mA of current. Our system behaves similarly to that of an RCL circuit in that when resonance is achieved the impedance is at a minimum. We are holding the voltage amplitude constant, usually between 20-30 V, and as we approach resonance, the impedance decreases leading to an increase in current. This follows from Ohm’s Law \( V = IZ \), where \( Z \) is the impedance of the circuit. Previous research on our system found the impedance to be approximately 100 \( \Omega \) at resonance. Using this as our \( Z \), \( I = \frac{V}{Z} \) tells us that our transducer will draw approximately 200-300 mA of current at a voltage amplitude of 20-30 V when we are near resonance. As mentioned earlier, the A-301 can only source 100 mA so in order to increase the supplied current we ran the output to our custom current amplifier. The current amplifier consisted of two transistors, npn/pnp, in a push-pull

![Custom Current Amplifier](https://example.com/image.png)

FIG. 5. Components of the custom current amplifier (black box).
configuration (FIG. 5). Powering the current amplifier are two HP E-3612A 0-120V DC Power Supplies. These power supplies were connected in series, with a ground between them, allowing for a maximum voltage swing of -60 V to +60 V. The output signal from the current amplifier, now with a current of about 200-300 mA, was then fed into our piezo transducer. These elements are apparent in FIG. 6.

C. Acoustic Levitation and The Microphone

For this experiment a CZ034 Condenser Microphone was used to measure the changes in pressure as a function of position, \( z \). The microphone was mounted to a Starret 752A-12 caliper modified to be a translation stage that had a resolution of 0.01 mm. The diameter of the CZ034 microphone is 9.7 mm and, at resonance, the 28 kHz soundwave has a spacing of about 6 mm between pressure nodes. We attached a 1 mm gauge hypodermic needle to the face of the microphone in an attempt create a more precise measuring device. The needle needed to be slender enough as to not perturb the standing wave around it. This modification was made under the assumption that the changing pressure in the needle would be proportional to the changing pressure at its opening. Our modified microphone, along with the precision of the translation stage, allowed us to monitor the pressure at specific positions with repeatable results. To avoid eliminate the required 5 V voltage bias, the output from the microphone went through a high-pass filter with

\[
 f_c = \frac{1}{2\pi RC} = 72 \text{ Hz}. \]

The filtered signal from the microphone was then continuously monitored on a Tektronix TDS 2004C Oscilloscope.

The CZ034 is a condenser microphone and requires a voltage bias, in our case +5 V from an HP E3610A DC Power Supply. Condenser microphones are parallel plate
capacitors in which one of the plates is light enough and acts as a diaphragm for incoming pressure waves. Parallel plate capacitors have a capacitance that is inversely proportional to the distance, \( d \), between the plates, \( C = \frac{\varepsilon_0 A}{d} \). The voltage across a capacitor is inversely proportional to the capacitance, \( V = \frac{q}{C} \). From here we can see that the voltage is directly proportional to the distance between the plates. This means that as the diaphragm moves \( d \) changes and when \( d \) changes the voltage changes. We can then monitor this change in voltage on an oscilloscope. At a pressure node the diaphragm will not feel a varying pressure so \( d \) remains constant and no voltage change is detected.

If the pressure wave intensity is too high, the output of the microphone will be saturated. In our case, saturation of the microphone occurred if the driving current of the transducers exceeded 0.100 A which is approximately one-third of our typical driving current.
D. Schlieren Optics

The Schlieren optics setup was used to visualize the soundwaves and as an indicator of when resonance was achieved. Schlieren imaging allows for the visualization of any inhomogeneity in the air in the optical path. Our arrangement consisted of a 108 mm diameter concave mirror that had a focal length of 1.2 m placed behind our transducer. A point source of light, in our case an LED with an iris, was placed facing the mirror at twice the focal length, $2f$, of the mirror. With a point source of light at a distance of $2f$, the reflected image will also be at $2f$. The mirror equation explains this relationship.

$$\frac{1}{f} = \frac{1}{d} + \frac{1}{d'}$$

If $d'$ represents the location of the image and $d = 2f$ represents the position of the object, then the solution to the mirror equation

$$\frac{1}{f} = \frac{1}{2f} + \frac{1}{d'}$$

tells us that $d'$ is also $2f$.

The next step in the Schlieren setup is to place a light stop at the focused image such that the width of the stop is approximately the diameter of the focused light. Ideally a point source of light requires a point source light stop; however, a point source light stop is difficult to make so we opted for a thin hex wrench. The final piece, although this
element may be substituted for a well-attuned eye, is a camera. In this experiment we used a Sony DSC-Rx10M3 camera. If the air between the camera and mirror remains unperturbed, the light will remain blocked. Schlieren imaging works when there exist perturbations in the surrounding air. These perturbations cause a change in the index of refraction in the air and as a result the light’s path is deviated. Deviated light will go past the light stop and be recorded in the camera. Adjusting the size of the light stop will adjust the sensitivity in detecting deviated light; but, allowing too little or too much light will prevent detection of perturbations. If the light stop allows for too much light to enter the camera, the resulting image will be washed out and deviated light will go unnoticed. If the area of the light stop is much greater than the point source of light then a majority of the deviated light will also be blocked and the resulting image, if any, will be faint.
IV. THE EXPERIMENT

A. Determining Resonance

Acoustic levitation depends almost entirely on resonance. In this experiment we are most concerned with two areas of resonance: the resonant frequency of the transducer, and the resonant spacing between the transducer and the reflector. To efficiently maintain levitation we needed a convenient means of finding both forms of resonance.

1. Transducer Resonance

To determine our most efficient indicator of resonance, within the transducer, we considered a single driving transducer and three different methods that could signify resonance. The first method is the most fundamental and involves monitoring the output signal of our microphone, which is suspended just above the transducer. No reflecting surfaces were implemented in this test. Without reflectors there will not be a standing wave; therefore, there will not be any dependence of the signal on the microphone’s distance from the transducer, aside from the distance being too far and no pressure change detected. If the driving amplitude of the transducer is held constant, then we expect to see a maximum response in the microphone at the resonant frequency. This is because a resonant frequency within the transducer corresponds to a maximum displacement of its surface and this maximum displacement yields a maximum pressure change; our microphone responds to pressure changes. To test this with our microphone we swept through a frequency range of 27.85 kHz to 28.15 kHz with a transducer driving amplitude of ≈1.0 Vpp. Recall that lower driving amplitudes are used in conjunction with
the microphone as to not saturate its output. We used this frequency range because the nominal frequency, as per the distributor, of the transducer is 28 kHz. To perform this portion of the experiment we wrote a LabVIEW program that remotely controlled our Agilent waveform generator and remotely monitored our Tektronix oscilloscope. The waveform generator was instructed to sweep through our frequency range in increments of 10 Hz. At each increment the program would then wait 1.00 seconds; this pause gave the transducer time to stabilize at each new frequency. After the stabilization period, the program then retrieved the amplitude of the microphone signal from the oscilloscope. Once the microphone amplitude was obtained, the current frequency and its

![Graph of microphone output vs. frequency](image)

**FIG. 9.** Graph of the measurements of microphone output versus frequency. Maximum output, i.e. resonance, occurs at 28090±5 Hz.
corresponding microphone output were written to an Excel file and then the waveform generator was instructed to increase the frequency again. After the amplitude was recorded at every frequency, we plotted frequency versus normalized microphone amplitude. A normalized amplitude is sufficient as we are only concerned with where the maximum occurs. The graph of this data is shown in FIG. 9 and from its analysis we can see that the frequency corresponding to maximum amplitude is 28090±5 Hz. This is in good agreement with our nominal value of 28000 Hz. However, not only would this method require leaving a microphone in the space for levitation, it would also require inserting the reflector after our resonant frequency was found. This is because we made these measurements without the reflector. Despite the microphone’s ability to detect resonance, having to place the reflector and remove the microphone each time resonance is achieved is very inefficient. Another means for finding resonance must be explored.

The second method for determining resonance stems from the assumption that our transducer system behaves similar to an RCL circuit. An RCL (resistor-capacitor-inductor) circuit has the property that if the current and driving voltage signals are in phase with each other, then the circuit is at resonance. This is due to the nature of capacitors and inductors. Capacitors try to maintain voltage and inductors try to maintain current. If the current and voltage are in phase, then the capacitor and inductor will be changing together and will not oppose the other’s behavior. While we don’t explicitly have an RCL circuit, certain elements of our system behave in a manner consistent with one. The resistive element in our circuit comes from a mix of actual resistors and the internal resistance of the amplifiers. The capacitive element can be derived from the construction of our piezos. The piezos are constructed with a conducting material on either side of a piezoelectric crystal. This arrangement creates parallel-plate capacitor in
each piezo. The *inductive-like* element comes from the motion of the transducer. In a
typical inductor the energy used to maintain current is stored in the magnetic field of the
inductor; in the case of our vibrating transducer, the energy used to maintain the current
comes from its kinetic energy. When the driving voltage of the transducer changes, the
momentum of the transducer will temporarily oppose this new driving signal, mimicking
the behavior of an inductance.

To test this RCL approximation we monitored the phase between the current and
driving voltage. We used our oscilloscope to monitor the current by measuring the
voltage across a precision 1.00 \( \Omega \) resistor. Another channel on the oscilloscope
monitored the voltage from the function generator. The oscilloscope then displayed the
phase between the two inputs. Using the same LabView program and frequency range as
the first method, we measured the phase between the current and voltage (I & V) every
10 Hz. Note that we are measuring the phase *difference* so we are looking for the
frequency at which we measure a value of zero. This data was then plotted as phase
versus frequency. From the analysis of this data (FIG. 10) we found that a phase
difference of zero occurred at a frequency of \( \approx 28085 \) Hz. This value is very similar to
that of the microphone and is also in good agreement with the accepted value of 28000
Hz. These results are also consistent with our system behaving similar to that of an RCL
circuit. This phase method for determining resonance is much more convenient than that
of the microphone, primarily because we can leave in the reflector; however,
continuously monitoring the small screen on the oscilloscope is not ideal.
The third method for determining resonance is more so a consequence of the second method. In the second method we discuss our circuit’s similarity to that of an RCL circuit. Another property that RCL circuits have is that the impedance ($Z$) of the circuit is a minimum at resonance. From Ohm’s Law, $V = IZ$, we know that if $Z$ is minimum then $I$ is a maximum. That means that for a given driving amplitude the frequency at which we draw a maximum current is the resonant frequency. To test this we again used our same LabView program for sweeping through frequency, but this time

FIG. 10. Graph of the phase difference between current and driving voltage. Zero phase occurs at $\approx 28085$ Hz.
the program also returned the current as measured by the oscilloscope. We then plotted normalized current amplitude versus frequency. Again using normalized amplitude as we only care about \textit{where} the maximum occurs. From this test we found that our maximum current amplitude, for this driving amplitude, occurred at 28080±5 Hz. This frequency is again consistent with not only our accepted value, but also our previous two resonance tests. To further speak to the agreement of the three methods we plotted the data from all

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{current_amplitude_vs_frequency.png}
\caption{Graph of the current amplitude versus frequency. Maximum current occurs at a frequency of 28080±5 Hz.}
\end{figure}
three on the same graph (FIG. 12). It is clear from FIG. 12 that all three methods are sufficient in determining resonance within the transducer.

Now all that is left to consider is the ease of use of each method. We previously stated that the microphone was impractical due to the removal of the reflecting surface so we really need only consider the monitoring of phase or current. This decision comes down to preference as they are both nearly the same. To monitor the phase we must keep an eye on the oscilloscope and its small screen; however, we only need to see a phase of zero to know we are at resonance and this may be easier than finding a max current. The current through the transducer is boldly displayed on the large screens of the E3612A.
power supplies. Having the current being displayed in such a large manner makes it
easier to determine its value at a glance. The downside is that we have to scan through
some range to determine a maximum. Our preference was to use the current monitoring
method to determine resonance for the rest of the experiment.

2. Resonant Spacing

Our second form of resonance referred to the distance between our driving
transducer and the reflector; should this spacing be an integer multiple of half of the
driving wavelength (in air) we called it a resonant spacing. A resonant standing wave can
be set up between our transducer and reflector regardless of the driving frequency so long
as we have a resonant spacing of that frequency. Much like our transducer resonance we
needed a convenient means of locating these resonant spacings.

Maintaining levitation requires a sustained resonant spacing. We initially used the “by
eye” method to determine this spacing. The “by eye” method relied on coarse
adjustments to the spacing between the transducer and the reflector until we saw an
otherwise unlevitated object become perturbed or even display unstable levitation. When
the object displayed these signs we knew we were close to a resonant spacing, as once we
reached one the object should levitate; therefore, when these perturbations occurred we
made finer adjustments to dial in the resonance. To make these fine adjustments we used
a ThorLab’s translation stage that travels 0.6 mm per revolution and has a total travel of
30 mm. With the speed of sound in air being 340 m/s we know that within our frequency
range, 27.5 kHz to 28.5 kHz, a half-wavelength measures ≈ 6 mm; therefore, once we had
a resonant spacing we knew we were ≈6 mm from the next one.
While the “by eye” method proved to be incredibly straightforward, especially after a resonant spacing was found, it had the minor drawback of needing to observe an object while adjusting the spacing. Much like our internal resonant frequency we wanted some sort of quantitative indicator for spatial resonance. To accomplish this, we thought of using our piezos as detectors for resonance; more specifically, the piezos within an idle transducer. As previously discussed, a piezo will expand and come to rest when a voltage is put across it; conversely, a piezo will also produce a voltage when a varying pressure is applied to it. A maximum voltage signal should be produced by the piezo when a maximum pressure change, i.e. a pressure antinode, is applied to it. We know that when we achieve a standing wave we are at a resonant spacing and the surfaces of the transducer and reflector are pressure antinodes; therefore, should a resonant spacing be achieved, our piezo detector (an idle transducer) should yield a maximum signal output.

To test this we attached a probe across the terminals of an idle transducer that is

![Image of equipment setup](image-url)

**Fig. 13.** Photograph of the minimum amplitude of the signal output of our piezo detector. Note that the Styrofoam spheres are not levitating.
serving as a reflector. We then monitored the output signal of the probe on our
oscilloscope and noted various minima and maxima of the signal amplitude. At a given
minimum amplitude we had no success in levitating. Figure 13 depicts the typical signal
amplitude and results for attempted levitation at a piezo output minimum. For a given
maximum piezo output amplitude we consistently had stable levitation; a typical example
of this situation is in FIG. 14.

![Image of experiment setup]

FIG. 14. Photograph of successful levitation at the location of maximum detector signal
output.

From this data we concluded that using a piezo output signal is an efficient
method for determining a resonant spacing. Being able to find a resonant spacing without
needing a levitated object is the primary benefit of this method. Outside of our
experiment this benefit would especially be relevant in the case of levitating a highly
reactive substance or small lifeform that might run away. A third means for determining
resonant spacing, not utilized until later in the experiment, was Schlieren imaging. This
will be discussed in a later section but the results of this method concurred with our piezo detector method.

With the success of the piezo detector method, we now had efficient and accurate processes for determining our mechanical transducer resonance and the accompanying resonant spacing. For the transducer system we will be using the maximum supplied current as an indication of the resonant frequency of the transducer and for the resonant spacing of that frequency we will be using the position of the reflecting transducer that yields a maximum piezo output signal.

B. Resonance Behavior

Throughout the resonance tests we noticed inconsistencies in the internal resonant frequencies of the transducers and while they were not significant enough to impact our results from the tests, the behavior of the transducers needs to be well understood to maintain stable levitation. Nominally the transducers have a resonant frequency of 28.0 kHz, stemming from their mass and length as previously mentioned, but during our resonance tests we noticed a large range of frequencies, about 27.2 to 27.9 kHz. To analyze the cause of this deviation we looked at the sources of variability within our system.

Of the active components in our system—current/voltage amplifier, power supplies and waveform generator—the most probable cause of variation comes from the changing of parameters for the function generator. This was deduced from the fact that our power supplies are very stable and that the amplifiers depend on the signal from the generator. Within the waveform generator there are only two variables contributing to our
signal. The first was the driving amplitude of the signal and the second was whether we were increasing or decreasing in frequency. We concern ourselves with increasing or decreasing frequencies because we suspect there may be a hysteresis-like effect in the transducer behavior.

Outside of our system components we found the only source of variability to be the change in temperature of the transducer; as a transducer was being driven the mechanical oscillations caused it to heat up. Having identified the waveform driving amplitude, increasing or decreasing frequency and the heating of the transducer as likely causes of deviation in resonance, we next wanted to determine how these effected the resonance.

1. Effect of Driving Amplitude and Frequency Direction

To analyze these effects on resonant frequency we monitored the current through the transducer (a proxy for resonance) as a function of frequency for a range of driving amplitudes. To make these measurements we again used a LabVIEW program. For a given driving amplitude our program swept through a driving frequency range of 27.3-28.1 kHz in increments of 10 Hz. We swept through this range twice with data collection ending after each sweep. The first sweep started at 28.1 and decreased to 27.3 kHz and then the next sweep started at 27.3 and increased to 28.1 kHz. When the driving frequency of the transducer was set, the program then waited 1.00 second for stabilization before acquiring the current at that frequency. This program is almost the same as the one used in the determining resonance section; the difference is that after the current had been measured for each frequency in our range, the program increased the driving amplitude.
The current through the transducer as a function of increasing or decreasing frequency was measured over a driving amplitude range of 0.5-3.0 Vpp from the function generator; this data is shown in FIG. 15 for the decreasing case.

From FIG. 15 it is evident that the resonant frequency of the transducer will decrease with an increase in driving amplitude for a decreasing driving frequency. Within our driving amplitude range our resonant frequency shifts by approximately 400 Hz. To better understand the behavior of the resonant frequency we plotted resonant frequency vs. driving amplitude. The relationship between them is nearly linear and we can approximate the behavior of the resonant frequency by fitting a line to this data. This plot
(FIG. 16) allows us to predict the resonant frequency at a given driving amplitude when we are decreasing in frequency. The slope from FIG. 16 indicates that the resonant frequency will decrease by \( \approx 157 \) Hz for a 1.0 Vpp increase in driving amplitude. The intercept of the line from FIG. 16 tells us that when the transducers are being driven with an amplitude of 0.00 Vpp, i.e. they are idle, the resonant frequency is \( \approx 27.890 \) kHz; this value is consistent with the nominal resonant frequency of 28.00 kHz.

We then repeated these tests for the increasing frequency case. The overall behavior was the same in that an increase in driving amplitude yields a decrease in resonant frequency, but the main difference is that the range of resonant frequencies is
Our measurements for the increasing case are seen in FIG. 17. For the increasing case we see that the lower bound for our resonant frequencies is $\approx 27.550 \text{ kHz}$ and this gives a total shift in resonance of $\approx 300 \text{ Hz}$. This difference in the behavior, based on an increasing or decreasing frequency sweep, is consistent with the “frequency response hysteresis” as mentioned in the Langevin-type transducer article.\(^5\) To get a better understanding of this data we again graphed resonant frequency as a function of driving amplitude (FIG. 18). The fit of this data tells us that, for the increasing case, a 1.00 Vpp increase in driving amplitude will decrease the resonant frequency by $\approx 117 \text{ Hz}$. 

\[ \text{FIG. 17. Graph of Current amplitude vs. Increasing Frequency vs. Driving amplitude shows a decrease in resonant frequency for increasing driving amplitude. Note the lower bound for resonant frequency is greater than in the decreasing case.} \]
This drift in resonance is 20% less than in the decreasing case and gives a much narrower resonant frequency range.

In both cases we see an overwhelming trend of a lower resonant frequency at higher driving amplitude. Another commonality between these two cases is that we see several substantial drop-offs in current amplitude at the higher driving amplitudes. We still are unsure as to the exact cause of this drop-off, but as long as we stay on the high side of the resonant frequency then we should maintain a stable current amplitude.

To minimize this drop off, and ensure maximum current stability, we decided to use the decreasing case to find resonance at a given driving amplitude. This way we would not come across any drop-offs on the way to the resonant frequency and our newly

FIG. 18. Resonant frequency vs driving amplitude for an increasing driving frequency. The slope of this graph gives the change in resonance for a 1.00Vpp change in driving amplitude.
mapped behavior at each driving amplitude will tell us when we are close to resonance allowing us to always stay on the high side of resonance.

2. Effect of Heating

Throughout our experiment we operate at a relatively constant driving amplitude, 2.00-3.00 Vpp from the function generator, so the maximum drift in resonance, caused by driving amplitude, is usually 150 Hz. In addition to this dependence on driving amplitude, we have also noticed a drift in resonance at a constant amplitude. We suspect that this drift at constant amplitude comes from the heating of the transducer. During our work with the transducers up to this point we had noticed substantial temperature changes in them when operated for long periods of time. The high frequency motion of the transducers made it near impossible to reliably attach a thermocouple to monitor temperature over time so we used a more primitive method, our hands. This method proved consistent only for the extremes of our temperature range as it is difficult to feel subtle differences with our hands; however, knowing the extremes was sufficient as we primarily just wanted to gauge the order of magnitude of the resonant frequency shifts. The extremes, as determined by touch, were when the transducers were at equilibrium with the room and when we were unable to touch them for more than three seconds without risking burning ourselves. As a reference, the ideal cutoff temperature for this style of transducer is 150°C and while we didn’t reach that, they still became incredibly hot.

When a transducer was at rest for a significant period of time it was at approximate equilibrium with the surrounding air and when driven with a constant amplitude the resonant frequency at this time was slightly higher than predicted from our
earlier tests. Choosing to run the transducer at a midrange driving amplitude, \( \approx 2.50 \) Vpp, we noticed that at our high temperature limit the amplitude of the current had decreased. This indicated that we were no longer at a resonant frequency. When we found the resonant frequency at our high temperature limit we noticed that it was significantly lower than the low temperature frequency and the frequency predicted by our graphs of resonance vs. driving amplitude. On average the high temperature limit had a resonant frequency 200 Hz less than the low temperature and about 150 Hz less than our predicted frequency at that amplitude. From these trials it is evident that an increase in temperature also corresponds to a decrease in resonant frequency.

When we discovered this relationship between temperature and resonance we were concerned it may have inflated the effect of driving amplitude on resonance that we found earlier, as both have a downward trend. To circumvent this inflation, we repeated our earlier tests of current amplitude vs frequency vs driving amplitude except we now implemented a temperature change threshold. By this I mean, at the beginning of each sweep of the frequency range we ensured that the transducer was at a similar warm temperature. These repeated measurements differed negligibly from what he had found initially. The data in our earlier graphs, FIG. 15 and FIG. 17, is from these repeated measurements.

From our tests we found that an increase in temperature and an increase in driving amplitude cause the resonant frequency of the transducer to decrease. We concluded that under the conditions we operate in each of these can contribute a maximum downward shift of \( \approx 150 \) Hz. We also discovered that our transducers exhibit a drop-off in current when decreasing the frequency from resonance. With this in mind we can use our graphs
to determine a frequency plateau that will ensure stable current supply, i.e. stable levitation.

C. Single Transducer Levitation

The single transducer system is the simplest arrangement for acoustic levitation and it served as the primary setup during our levitation trials. In this arrangement a single driving transducer and a reflector established a resonant standing wave with which we could levitate appropriately sized objects, <6 mm diameter, at the pressure nodes. The restriction on size comes from the distance between antinodes, half of the wavelength, of our soundwave. We are using \( \approx 28 \text{ kHz} \) soundwaves, which in air at a velocity of 343 m/s have a wavelength of \( \approx 12 \text{ mm} \). During experimentation with the single transducer system we used our understanding of the transducer’s behavior as well as a few structural changes to investigate methods that increase the stability of a spherical levitated object; for example, Styrofoam balls. Once we achieved the desired stability we explored the effect of various perturbations on spherically symmetric objects and finally we experimented with levitating irregularly-shaped objects.

1. Optimum Stability

For this system we concerned ourselves with two areas of stability: the stability in the driving current and the stability of the levitated object. A stable current amplitude will ensure that we do not experience a sudden decrease in power and a stable object is unlikely to be thrown out of the pressure node. To maintain the stability in the driving current, we utilized a few of the key points from the previous section: as the transducer
heats up the resonant frequency decreases, for a given driving amplitude there is a relatively stable current plateau on the high-frequency side of resonance and if the frequency is decreased through the resonant frequency then the current drops off significantly (FIG. 15). With the current being dependent on the driving frequency of the transducer, we will be able to maintain current stability by maintaining the appropriate driving frequency. It is important to note that running the transducer at its resonant frequency is not necessary. We are more concerned with maintaining a high driving current and we know that near resonance we have a range of frequencies that produce these high currents (FIG. 19). To maintain this high current amplitude we again used a
LabVIEW program. With our transducer running at resonance at an initial temperature of about 40 °C, our program recorded the current driving-current amplitude as the maximum value. The program continuously monitored the driving current amplitude as given by the HP E3612A power supplies. If the current decreased by more than 5%, from the maximum value, then the program decreased the driving frequency until the current amplitude was again within 5% (FIG. 19). The program ran as long as we were levitating or until we decided to use manual adjustments.

There are a few important things to note about this process. The first is that it is not completely autonomous, if we change the driving frequency of the transducer then we must also change the spacing of the transducer and reflector to maintain a resonant standing wave. Changing the spacing requires adjusting the translation stage that the reflector is mounted on. Secondly, our program uses a 5% threshold in current amplitude to determine when it should communicate with the function generator and this is a tradeoff between efficiency and stability. If we wanted maximum current stability then we would decrease the driving frequency if the change in current amplitude exceeded the uncertainty\(^1\), ≈1%, in the measuring device; however, this is inefficient in that the resonant spacing would need to be adjusted much more frequently. Lastly, although we are not running the transducer at its resonant frequency we can adjust the resonant spacing so that any driving frequency can yield a resonant standing wave, we just want the intensity of that standing wave to be similar for varying frequencies.

With a stable current and a resonant standing wave established, the stability of a levitated object was related to the localization of the pressure node. In the

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\(^1\) This would indicate an unusual change in amplitude that was likely caused by a shift in resonance.
early models of our experiment the transducer and reflector had planar, parallel surfaces. This proved to be a problem as our pressure node became a nodal plane, so at each pressure node along the z-axis our object was able to move freely in the $xy$ plane. At times the object would move violently enough in this plane that it would be thrown outside of the system. Reducing this planar symmetry allowed for a more consistent, localized pressure node resulting in much more stable levitation. The first approach we used was to reduce the intensity of the pressure waves near the edges of the transducer. This will give a more centralized column of pressure waves. To do this we attached a soft foam ring to the surface of the transducer (FIG. 20). The diameter of the transducer is 6.6 cm and the foam ring has inner and outer diameters of 4.0 cm and 6.6 cm respectively. We experimented with different inner diameters to try and minimize the effect of the planar symmetry. During these trials we found that too small of an inner diameter, <4.0
cm, compromised our ability to levitate because too much of the pressure wave was blocked by the foam.

To further increase the stability we opted for a shallow, concave reflector in hopes that it would provide a focusing effect. Our initial curved reflector was a petri dish with a similar diameter to that of our transducer and a radius of curvature of 14.5 cm (FIG. 21). In choosing the radius of curvature we wanted a reflector that had a focal point within our usual spacing of the transducer and reflector. The relationship between the radius of curvature of a spherical reflector and its focal point is

\[ f = \frac{R}{2} \]

Our typical reflector/transducer spacing is 5-9 cm and the focal length of our petri dish is 7.25 cm. The petri dish was glued to the surface of an idle transducer as a temporary means of testing its effect on stability. During the use of the petri dish we examined the stability of our object with and without the foam ring and we noticed the ring had a negligible effect on the stability. With this in mind we opted to not use the ring and noticed that the particle exhibited a high amount of stability along the entire z-axis, not just at the focal point. We decided to manufacture a more permanent and rigid reflector by machining the surface of one of the transducers. The machined transducer has a radius of curvature of 13.2 cm with a corresponding focal length of 6.6 cm. Figure 21 demonstrates the z-axis stability for several levitated Styrofoam balls. The driving
frequency of the trial in FIG. 20 was ≈27.8 kHz and we measured the distance between the center neighboring balls to be ≈6 mm. Using \( c = \lambda \nu \) we can estimate the speed of sound in air to be 340 m/s, which is in agreement with the accepted value we are using, 343 m/s at 20 °C.

2. Damped Oscillator

With the implementation of our LabVIEW program and our curved reflector we managed to produce consistent, stable levitation of our Styrofoam balls. From here we wanted to extend our levitation trials in an attempt to characterize the force and pressure amplitude on our Styrofoam ball. In a previous section we derived Eq. (30) for the force on a sphere in a standing pressure wave. The force acting on the sphere has a \( z \)-component,

\[
F_z = \frac{-5\pi A^2 k R^3}{6\rho_0 c_0^2} \sin(2kZ). \tag{31}
\]

Where \( A \) is the pressure wave amplitude, \( R \) is the radius of the sphere, \( \rho_0 \) is the density of the air, \( c_0 \) is the speed of sound in air, \( k = \frac{2\pi}{\lambda} \) = the wave number, and \( Z \) is the distance away from the pressure node. We can set the location of the pressure nodes to be \( Z = 0 \). It is important to note that the force acts like a spring to push the sphere towards the node, so to first order we can approximate the force as that of a harmonic oscillator. If we take \( Z \) to be 0.5 millimeters, which is much larger than the usual case, then the
product \((kZ)\) is much less than one. With this in mind we can approximate the force on the sphere to be

\[
F_z \approx \frac{-5\pi A^2 k^2 R^3}{3\rho_0 c_0^2} Z. \tag{32}
\]

If we liken this force to that of a mass on a spring, \(F = -SZ\), it can be said that our spring constant, \(S\), of the acoustic levitation force is

\[
S \approx \frac{5\pi A^2 k^2 R^3}{3\rho_0 c_0^2}. \tag{33}
\]

FIG. 22. Damped oscillations of a Styrofoam ball in an \(\approx 28\) kHz standing pressure wave. A region of small amplitude, such that \(kZ \ll 1\), was fit to a damped oscillator equation and yielded a natural frequency of \(\omega_0 = 556 \pm 1\) rad/s.
The spring constant can also be related to the natural frequency, \( \omega_0 \), and the mass of the sphere with 
\[
S = m\omega_0^2.
\]

To find the natural frequency we will perturb the Styrofoam ball while it is levitating and treat the resulting oscillations as we would a damped oscillator. For repeatable perturbations of the sphere we used a square wave pulse modulation to turn off the transducers for \( \approx 23 \) ms every 1.6 seconds. This modulation allowed for a temporary free fall of the sphere followed by several oscillations as the sphere was pushed about the node until it finally settled. To record these oscillations we used a 20x slow motion capture with our Sony RX10 III camera. Using Graphical Analysis we measured the position versus time for each frame of the recording. Fitting this data to the damped harmonic oscillator equation, 
\[
Z = Z_0 + Ae^{-\beta t}\cos\left(\sqrt{\omega_0^2 - \beta^2}t - \delta\right),
\]
where \( \beta \) is the damping coefficient, we can find \( \omega_0 \) and determine \( S \). Note that if \( \beta = 0 \) we would have the equation of an undamped harmonic oscillator.

Figure 22 displays our measured data- black dots- and our fitted curve, the red line. Note that the first several points on the graph show the Styrofoam in free fall. Only a region of small amplitude, \( Z \approx 0.5 \) mm, was used for the fit as to be consistent with our approximation from earlier. The fit from this region returned a natural frequency of \( \omega_0 = 556 \pm 1 \) rad/s. Now that we have \( \omega_0 \), and we measure the mass of the Styrofoam to be \( m = 2.80 \times 10^{-7} \) kg, we can calculate \( S \).

\[
S = m\omega_0^2 = 0.0866 \frac{N}{m}
\]
With a value for $S$ determined, we were then able to determine an approximation for the pressure wave amplitude. Solving for $A$ in our spring constant equation will give the following result:

$$A = \sqrt{\frac{3S \rho_0 c_0^2}{5\pi R^3 k^2}}. \quad (34)$$

Recall that $\lambda = \frac{c_0}{f}$, where $f$ is the frequency of our sound wave, and with this we can rewrite the equation for $A$ as:

$$A = \sqrt{\frac{3S \rho_0 c_0^4}{5\pi R^3 (2\pi f)^2}}. \quad (35)$$

To get an estimate of $A$ we will take $\rho_0 \approx 1.20 \text{ kg/m}^3$ (at $\approx 20^\circ C$), $c_0 \approx 343 \text{ m/s}$, $f = 27.9 \text{ kHz}$ and $R \approx 1.20 \times 10^{-3} \text{ m}$. Using these values we determined $A \approx 2270 \text{ Pa}$. If we compare our pressure amplitude to air at STP, $1.01 \times 10^5 \text{ Pa}$, we can see that our pressure amplitude is about 2.2% of the atmospheric pressure.

We then used Newton’s second law to get an approximation for the force and the $Z$ displacement. When the Styrofoam ball is levitating unperturbed we know that it is not moving in the $Z$ direction, so the net force on the Styrofoam is zero. Therefore, Newton’s second law becomes

$$F_z - m_{ball}g = 0 \rightarrow F_z = -SZ = m_{ball}g = 2.75 \mu\text{N}. \quad (36)$$

Recall that this force is pointing toward the pressure node, which in this case is in the +Z direction.

We can also use Newton’s second law to get an estimate of our displacement from the node. Solving for $Z$ in Eq.(36) we get
This means that our Styrofoam ball is approximately 32 microns below a node. 32 microns is much smaller than our original assumption of 0.5 millimeters, which means that our approximation of $kZ \ll 1$ still holds true.

3. Levitation of Liquids

The pressure nodes of our standing wave act as regions of microgravity and if we suspend a liquid in these regions then the surface tension will cause the liquid to form a sphere. We wanted to explore the behavior of these liquid spheres and examine how similar they were to our Styrofoam balls. The two liquids we considered were isopropanol and water. The densities of these liquids are 0.79 g/cm$^3$ for isopropanol and 1.0 g/cm$^3$ for water. These are much more dense than the Styrofoam balls, 0.04 g/cm$^3$. With this in mind we found that a higher driving voltage, $>2.5$ Vpp from the function generator, was needed to maintain levitation with this increase in mass. The higher driving voltage allowed the transducer to draw currents in excess of 0.25 A, which we used as a minimum for our liquid levitation. This differed greatly from the Styrofoam balls that were light enough that we were able to maintain levitation at a driving voltage of 0.4 Vpp and a current of 0.05 A.

Objects with a lower density remain closer to the equilibrium points so they will exhibit more stable levitation. Using Eqs.(32) and (36) we can derive a relationship between the density of a spherical object and its displacement from an equilibrium point. First we will express the force per volume using Eq.(32) and the volume of a sphere
\[
\frac{3F_z}{4\pi R^3} = \frac{F_z}{V_{\text{object}}} \approx \frac{-5A^2 k^2}{4\rho_0 c_0^2} Z.
\] (37)

We can then rewrite Eq.(37) using Eq.(36) to get the relationship between the density of the object and the displacement from the node

\[
\frac{F_z}{V_{\text{object}}} = \frac{m_{\text{object}} g}{V_{\text{object}}} = \rho_{\text{object}} g \approx \frac{-5A^2 k^2}{4\rho_0 c_0^2} Z.
\] (38)

Finally, if we solve Eq.(38) for \(Z\) we can see that an increase in the density of the object will make the object levitate further below the node

\[
Z = -\rho_{\text{object}} \frac{4g \rho_0 c_0^2}{5A^2 k^2}.
\] (39)

Eq.(39) is a linear relationship between displacement and density. Water’s density of 1.0 g/cm\(^3\) is 25 times that of our Styrofoam ball, which means that the water drop should hang 25 times lower, 25x(−31.9x10\(^{-6}\) m) = −0.80 mm. With the center of a 1.3 mm radius water drop 0.80 mm below the node, we can see that the bottom edge of the drop is at a position 2.1 mm below the node. At this distance below the node we start to see some instability in the levitation. For our system we can only levitate objects up to 6mm in diameter and even at the 5 mm size we still notice some instability.

With the isopropanol having a density between that of our Styrofoam and water, we used it as an intermediate step in our levitation trials. Isopropanol also has a much lower surface tension than water and this was both advantageous and a significant drawback. The advantage of the lower surface tension is that if we dropped the isopropanol on the surface of the driving transducer, the vibrations of the transducer would vaporize the liquid. This vapor would then rise and then condense at our various nodal points. This was an advantage because it made loading the vapor into the nodes very easy although we were not guaranteed that the vapor would condense at the same.
points with every trial. Aside from issues involving consistent location of the liquid isopropanol sphere, the main disadvantage of the isopropanol was that it evaporated quickly. Nonetheless, we had managed to consistently levitate the isopropanol in a stable manner and with this accomplished we then focused our attention to water.

Water’s high surface tension prevented us from using the same loading method as we did with the isopropanol, instead we used a hypodermic needle. The needle allowed for precision loading of the water drops making repeatability easy. With this needle we were able to levitate multiple water drops (FIG. 23) at given instance much like with our Styrofoam. Over the course of many trials we noticed that the average water drop had a radius of 1.3 mm. We found this average by levitating a similarly sized Styrofoam ball in the adjacent nodes and measuring the radius of said ball.

When levitating water droplets we were particularly interested with inducing normal mode oscillations in them. From a paper written by Ran et al., the resonant frequencies of a water drop are given by

$$f_n = \frac{1}{2\pi} \sqrt{\frac{n(n-1)(n+2)\gamma}{\rho R^3}}$$  \hspace{1cm} (40)$$

where $n$ is the harmonic, $\gamma \approx 70\times 10^{-3}$ N·m$^{-1}$ is the surface tension of water, $\rho = 1000$ kg/m$^3$ is the density of water and $R$ is the radius of the drop. Eq.(40) is very
sensitive to changes in $R$ so we found the $n=2$ harmonic for a range of $R$ in case our estimate of $R=1.3$ mm wasn’t adequate. For a water drop with a radius between 0.9 and 2.0 mm we found that the second harmonic frequency is in the range 40-140 Hz. To induce the second harmonic oscillations, we modulated the amplitude of our driving signal at frequencies spanning 40-140 Hz. With modulation frequencies in this range we were unable to excite the normal modes for a significant time. In some instances, we witnessed what appeared to be normal oscillations but they were brief and followed by the water falling out of the node. It would appear that our system can only sustain levitation of an unperturbed water drop at this time.

4. Non-Spherical and Asymmetric levitation

The majority of our experiment investigated the levitation of spherically symmetric objects but we also wanted to examine the behavior of some irregularly shaped objects; a few of these were balsa wood, pieces of paper, and an ant. All of these exhibited stability along the $z$-axis; that is, they didn’t bounce up and down within the node. However, we did witness a large amount of rotation in the $xy$ plane and this was due to the object lacking rotational symmetry. The most interesting of these trials was that of an ant. A live ant was chilled on a piece of ice to make it easier to handle. From here we used a pair of tweezers to gently place the ant within the node. Levitation of the ant was

![FIG.24. Live ant levitated and then released (still alive).]
stable, with the exception of rotation in the $xy$ plane, and after we released the ant it did not exhibit any negative side effects from the pressure wave.

D. Schlieren Imaging Analysis

Schlieren imaging allows for the visualization of perturbations in the air along a certain optical path. We used a Schlieren optics arrangement as a means to visualize the standing pressure wave that produced our levitation force. Using our Sony RX10 III camera we were able to produce a live feed of our pressure wave as well as capture images of it (FIG. 25). The live feed of the Schlieren was useful in determining our resonant spaces. When the spacing between our transducer and reflector was not a resonant spacing, we were unable to see the standing wave pattern on the camera. In examining the standing wave, we can see light and dark bands, which correspond to our pressure nodes and antinodes. Our initial assumption was that the light bands correspond to the pressure antinodes and we had two reasons to believe this. The first was that Schlieren imaging displays perturbations in the air and the locations of our largest perturbations are the pressure antinodes. The second reason was that the dark area outside of the standing wave is not
undergoing any change in pressure; therefore, the dark bands inside the pressure wave should also be positions that are not experiencing changes in pressure, pressure nodes. However, when we levitated the Styrofoam balls we noticed that they remained in the light bands and this contradicted our theory of the pressure nodes being positions of levitation (FIG. 25).

To determine whether the light bands were nodes or antinodes we used our microphone to measure the change in pressure as a function of distance away from the driving transducer. We took measurements in \( \approx 1.2 \) mm intervals with additional measurements made at the center of the light bands. Our data was then graphed and fit to a sine squared function (FIG. 26). The parameter \( w = 6.4 \) mm from the fit corresponds to the distance between the pressure nodes, and is consistent with the value of 6 mm that we
have been using. From our data we see that the change in pressure is consistently a minimum at the light bands. We are unsure as to why the pressure nodes are illuminated by the Schlieren but we trust our measurements, and the theory Eq.(30), when it comes to concluding that the Styrofoam balls levitate at pressure nodes.
V. CONCLUSION

In this experiment we first analyzed the behavior of commercial, ultrasonic transducers. We found that the resonant frequency of the transducers decreased with an increase in temperature and the driving voltage amplitude. Without the effects of heating, the range of possible driving amplitudes used in this experiment, 0.5-3.0 Vpp from the function generator, caused the resonant frequency to vary from 27.9-27.5 kHz respectively. The vibrations of the transducer caused it to heat up significantly, up to \( \approx 80^\circ C \), causing an even more dramatic decrease in the resonant frequency. We also found that the transducers exhibited a frequency response hysteresis; that is, if you decrease the frequency through resonance, \( \approx 28 \) kHz, an immediate increase back to 28 kHz will NOT yield a resonant frequency, as determined by a maximum in driving current. An increasing or decreasing frequency sweep will also effect the shift in resonance due to driving amplitude. An increasing frequency gives a narrower range in resonant frequencies, which would seem to be ideal; however, the decrease in resonant frequency caused by heating makes using a decreasing frequency sweep more sensible. Moreover, a decreasing frequency sweep ensures that we stay in a region of relatively high current amplitude.

In the next part of the experiment we used a transducer and a curved reflector to establish a standing pressure. By time averaging nonlinear terms in the acoustic pressure we were able to derive an expression for the force on a spherical object from this pressure. This force is called the acoustic levitation force and it is a restoring force about a pressure node in the standing pressure wave. Objects can be levitated at this pressure node and through the dynamical behavior of these objects we calculated the amplitude of
our pressure wave to be \( \approx 2.2\% \) of 1 atm. We also showed the direct relationship between the density of a levitated object and its equilibrium displacement from a pressure node. We were able to confirm that the objects were levitating at pressure nodes by using a Schlieren optics arrangement and a microphone.

Future experiments may investigate the dual-transducer system as a means to eliminate the resonant spacing dependence.
VI. REFERENCES


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