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Effective Teaching in High School Mathematics

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I wrote this paper for my Senior Studies course in Secondary Teaching Methods. Some of the questions that guided my research included "What practices and instructional techniques have been proven effective in teaching mathematics", "How do these differ from the practices of effective teaching in general", and "How does the new emphasis on educational standards impact the use of these practices in the classroom." As a mathematics teacher, I had both professional and personal interest in the subject--I wanted to know and understand effective instructional techniques in mathematics and reflect on how I could apply them in my own classroom. It is my goal to teach math in a way that builds critical thinking/reasoning skills and encourages mathematical communication.

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Over the past decades, mathematics instruction has undergone a “reform” movement that emphasizes critical thinking, communication, and collaborative learning over rote memorization or application of formulas, procedures, and basic skills. Analogously, a new set of teaching methods focusing on these goals has been labeled “effective mathematics instruction.” These “new” teaching practices match my philosophy of teaching math very nicely. I view mathematics as a practical and useful applied science on the one hand, and a means for improving reasoning and critical thinking on the other. I have always felt that a major goal of math instruction is to develop students' analytical and logical skills in ways that can be generalized to other areas of life. It is easiest, however, to teach math in a manner that emphasizes memorization instead. Since my foremost personal goal is to keep myself from falling into that trap, and to use a variety of methods that scaffold and develop students' higher-order thinking skills, I am highly interested in what the literature says is "effective" math practice.

Some of the questions that guided my research were: What types of higher-level thinking are required by state and national standards? How should the curriculum be organized to foster these types of higher-level thinking? What specific teacher actions build students' higher-level thinking skills? In this paper I will discuss a number of specific teaching practices the research has deemed effective for math instruction, some of the similarities and differences between effective math teaching and general effective teaching, the impact of state and national standards on effective math instruction, and how I plan to incorporate these practices into my own classroom.

Effective Teaching Practices in Mathematics

The teaching methods and strategies that constitute effective teaching of mathematics depend on one's definition of "mathematics." If school mathematics is merely a collection of formulas, rules, and procedures that must be memorized and mastered, then many traditional teaching techniques like drilling, individual worksheet practice, and flashcards could be considered effective. However, the current definition emphasizes that mathematics is an integrated whole, a study of structures and the relationships between things, and a way to study and understand the world around us. The goal of teaching mathematics is changing too – now teachers need to help students develop the skills they will use every day to solve mathematical and non-mathematical problems, which include the ability to reason, to explain and justify ideas, to use resources to find needed information, to work with other people on a problem, and to generalize to different situations, as well as the traditional ability to carry out computations and procedures. Zemelman, Daniels, and Hyde (1998) describe the math teacher's goal as "help[ing] all students develop mathematical power." This mathematical power allows a student to feel that mathematics is personally useful and meaningful, and to feel confident that he or she can understand and apply mathematics (p. 89). The aim is that mathematically powerful students will not just be able to apply mathematics, but will develop mathematical and problem-solving "habits of mind," which they will use constantly (Stein 2001, p. 112).

There are a number of overarching principles that appear frequently in literature on effective math instruction. These include a problem-oriented curriculum that focuses on ideas before skills. Teacher actions that are effective include deriving concepts, using cooperative group work, encouraging frequent mathematical communication, and using multiple representations and multiple strategies.

A Problem-Oriented Curriculum

The National Council of Teachers of Mathematics (2000) Problem Solving Standard states that high school math students should be able to "build new mathematical knowledge through problem solving; solve problems that arise in mathematics and elsewhere; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving" (NCTM, Problem Solving, ¶ 1). It is true that memorizing a procedure or formula can enable a student to easily solve a certain type of problem given by the teacher – but this method breaks down when the problems grow more complex and unfamiliar. Since much of real life deals with complex and unfamiliar problems for which there is no solution manual, the NCTM says that "a major goal of high school mathematics is to equip students with knowledge and tools that enable them to formulate, approach, and solve problems beyond those which they have studied" (NCTM, Problem Solving, ¶ 5). This means that students need opportunities to develop their problem-solving skills in authentic situations – they need to "investigate questions, tasks, and situations…[and] create and apply strategies to work on and solve problems" that are suggested by the teacher or the class. These problems should relate to the students' personal experiences or to the real world, because such problems motivate students. For example, because "the need to make decisions based on numerical data permeates society," students easily see the power of probability and statistics when it is applied in a real-world situation (Zemelman et al., 1998, p. 91-3).

In order to effectively teach with a problem-solving focus, a teacher needs to carefully plan problems that will give students the maximum opportunity to hone their skills. This means the problems need to be complex enough to let students approach them from different angles, explore different strategies, reflect on their progress, and revise their methods. It also means the problems must be within the grasp of the students, because if they are too hard and the students are repeatedly unsuccessful, they will lose confidence in their problem solving abilities and their willingness to work on problems will not develop, or could even be destroyed. The teacher also needs to be "courageous," willing to take risks as the students take the problem in an unexpected direction, and judicious in deciding when students are generating productive ideas and when they should be steered in another direction. Additionally, the NCTM makes the point that to effectively teach problem-solving skills, teachers must "themselves have the knowledge and dispositions of effective problem solvers" (NCTM, Problem Solving, Teacher's Role).

Focusing on Ideas, Not Skills

Stein (2001) notes that in past decades, math teachers were "more concerned with students' rote use of procedures rather than with their understanding of concepts and development of higher-order thinking skills" (p. 115). This focus on skills has contributed to generations of Americans feeling that math is boring, static, and repetitive. It squelches students' natural imaginative thinking and discourages them from developing and using new problem-solving techniques. Zemelman et al. (1998) write that while knowing facts or procedures "without true understanding of the underlying concepts guarantees serious problems with learning other concepts," focusing on understanding mathematical ideas makes students "far more likely to study mathematics voluntarily and acquire further skills as they are needed" (p. 89-90). Focusing on the ideas gives students a strong foundation for learning new, related ideas. It also helps them to know when to apply particular skills or procedures, because they see the underlying reasons that these methods work.

Battista (2000) writes that "how students construct new ideas is heavily dependent on the cognitive structures students have previously developed" (p. 147). Therefore, effective mathematics teachers are aware of their students' mathematical thinking, and structure their teaching of new ideas to work with or correct those ways of thinking. Additionally, part of focusing on ideas is teaching metacognitive strategies— "classroom discussions should deal with what it means to make sense of a mathematical idea, how to make sense of ideas, and how to know when you have made sense of an idea" (p. 146).

Deriving Concepts

Doing mathematics and *documenting or proving* mathematics are fundamentally opposite processes. Battista (2000) notes that "a major and critical component of doing mathematics at all levels…is intuitive and empirical/inductive thinking…Unfortunately, these mathematical processes are hidden by the deductive proof format in which mathematics is recorded and traditionally presented" (p. 150). Mathematical ideas are formed through a process of analyzing problems, trying a number of strategies to solve them, evaluating the strategies' effectiveness, looking for novel strategies, and verifying that a particular strategy is valid. Textbooks and traditional mathematics instruction, however, only show students the last step; they present one strategy for solving a problem and perhaps prove why it works. If students never go through the process of deriving a concept for themselves, they will have a narrow understanding of that specific concept, and they will not have any opportunities to develop good problem-solving skills in general.

Traditional mathematics instruction, where teachers "explicitly tell their students the important ideas…with little or no emphasis on how those concepts were derived" and do not provide them with "opportunities to derive mathematical concepts and procedures through their own problem solving efforts," does not help students to become the critical and logical thinkers that we want them to be. Yet in American classrooms, concepts are stated by the teacher 78% of the time. In contrast, concepts are stated only 17% of the time in Japanese classrooms and 23% of the time in German classrooms (Stein, 2001, p. 116). In order to be effective, math teachers need to select problems that highlight concepts they want students to learn and to allow students to figure out these concepts for themselves by working on the problems. Of course, some concepts are highly complex or do not lend themselves to a problem-solving approach. When teaching directly, the teacher should still explain how the concepts were derived, rather than simply presenting a procedure without showing what thinking processes led to it, or why it works. If students do not obtain this background information, either by working on the problem themselves or by hearing and seeing a teacher's explanation, the new idea will not fit into their conceptual framework, and they will resort to memorizing rather than understanding.

Cooperative Group Work

Cooperative learning groups are effective in math, just as they are in other disciplines, but they must be implemented with care. Reynolds and Muijs (1999) write that some research shows whole-class instruction and teacher-led discussions to be the most effective mathematics instruction method for teaching basic skills. However, they note the effectiveness of group work for teaching higher-level thinking. Cooperative group work helps students to reflect on and talk about their own ideas and thinking, and it forces them to consider other students' ideas, which may be very different from their own (p. 281). It can even reduce math anxiety and help them to "overcome their insecurity about problem-solving because they can see more able peers struggling over difficult problems" (p. 282). Debbie Dicker, a 15-year veteran mathematics teacher at Highland Park High School, says she finds groups to be especially productive because they give the students more sources for support and help, and allow her to monitor individual students better: "With groups, each student has three or four people to answer his question, as opposed to me trying to answer 27 different questions" (personal communication, September 18, 2003).

Effective group work requires a lot of preparation – "it is insufficient to put students in groups and let them get on with it" (Reynolds & Muijs, 1999, p. 282). Battista (2001) notes that all students in a group must be "fully engaged as partners" for group work to be most effective (p. 146). To avoid having a few students do the work while others sit passively, teachers should give clear instructions, and make sure all students are aware of the group's goals and how they will individually be held accountable. It is also important that the task is appropriate for group work, that it is centered on an important idea or concept, and that the students are interested by it

(Grouws & Cebulla, 2000, ¶ 6). The level of the task should be challenging, but not beyond the students' ability, or they will give up quickly. Group work is often more effective for introducing a new concept than for reviewing old material (Reynolds & Muijs, 1999, p. 282). Closure is essential to group work – if the students do not arrive at the key conclusions or procedures, the teacher should bring it up. A whole class discussion following the group work is an effective way to provide closure (Grouws & Cebulla, 2000, ¶ 7). Groups should be used in conjunction with sessions of direct teaching and individual work time. Grouws and Cebulla (2000) note that it may be useful to have students work in collaborative groups after they have worked on the task individually (¶ 6), and Reynolds and Muijs (1999) assert that a mixture of whole-class and collaborative group teaching is the most effective (p. 283).

Frequent Mathematical Communication

The NCTM's Communication Standard for high school students states that students should be able to "communicate their mathematical thinking coherently and clearly to…others, analyze and evaluate the mathematical thinking and strategies of others, and use the language of mathematics to express mathematical ideas precisely" (NCTM, 2000, Communication, ¶ 1). Talking and writing about mathematics helps students to reflect on their own thinking and refine their ideas. The ability to communicate is best developed through practice, so effective math teachers provide many opportunities for students to communicate about mathematical ideas, in groups and as a whole class, orally and in writing. Effective math teachers "create a classroom environment of mutual trust and respect in which students can critique mathematical thinking without personally criticizing their peers" (Pugalee, 2001, ¶ 7). However, Stein (2001) warns that a "'culture of niceness' in which any criticism or disagreement is considered unsocial" can undermine good mathematical discussion – the teacher needs to teach students how to critique logic and reasoning according to standards set by the class, without attacking other students (p. 139). Mathematical communication should start at an intuitive and everyday level, and the teacher should help students to refine their language and make it more precise as they gain experience in mathematical communication (p. 135). One of the techniques effective teachers use to do this is "revoicing" – restating a student's unclear or imprecise statement in a more mathematical (yet still understandable) way, and allowing the student to agree or disagree that the restatement actually represents the student's original thought. This technique models effective mathematical communication for the students, but it also lets them to keep ownership of their ideas and construct personal meanings of concepts (p. 132).

Using Multiple Representations and Strategies

How mathematical ideas are represented is linked to communicating them. Effective math teachers represent concepts and show how to solve problems in more than one way to help all learners get the most out of mathematics instruction. Multiple representations help students to make "personal meaning" out of math concepts and give them the "opportunity to think in diverse ways" (Stein, 2001, p. 119). Battista (2001) notes that students should use formal arguments as well as demonstrations, drawings, and physical objects (p. 160). Geometry and measurement are best learned through hands-on experiences that involve actual shapes and taking or estimating actual measurements (Zemelman et al., 93), and computer programs like Geometer's Sketchpad can also be useful (Battista, 2001, p. 153-4). An effective math teacher chooses problems that can be solved in more than one way to improve "students' flexibility of thinking" (Stein, 2001, p. 121). Group work is effective partly because it allows students to see and try a number of methods for solving a single problem. It is also necessary for students to learn to evaluate representations and strategies and determine which is most appropriate in a certain situation. Effective teachers elaborate on students' personal representations, introduce them to "conventional mathematical representations," and help their students to make connections between the two (NCTM, 2000, Representations, Teacher's Role, ¶ 1).

General Effective Teaching

There are some differences between general effective teaching and effective math instruction. Stein (2001) argues that the "general effective teaching practice" of starting a lesson with teacher explanation and modeling of a concept conflicts with “reform” mathematics best practice, which holds that students should explore new concepts on their own in groups (p. 138). Stein (2001) also notes that while general effective teaching practices emphasize preparing key questions ahead of time, the dynamic nature of effective math practice guarantees that lessons will go in unplanned directions, and to build on student contributions a teacher must be willing to abandon preplanned questions and develop new ones during the lesson. Therefore, the effective math teacher spends more planning time anticipating alternative ways the students may react to a problem, and less time planning specific questions to ask them (p. 140). Some of the best math practices are simply more specific versions of the general practices. For instance, effective communication forums in mathematics classrooms can require more setup than do classrooms in other disciplines. This is because the teacher and students not only have to create a supportive climate, but they need to establish ground rules about what makes an adequate explanation, “who has the right to question and challenge mathematical solutions, and the basis on which claims and counterclaims will be judged” (Stein, 2001, p. 134, 139). Certainly mathematics instruction is more effective when the teacher applies general best practices like providing high opportunity to learn, communicating high expectations, making smooth transitions, having good classroom management, using a variety of modalities and techniques, and making sure students are engaged with the material. However, if effective math instruction techniques are used, they will imply most of these general practices.

The Impact of Standards

State and national standards for mathematics programs have had an enormous impact on what is considered effective mathematics instruction. Because the NCTM standards and Illinois Learning Standards now require math students to demonstrate reasoning, communication, and problem-solving ability, practices that were once considered effective because they increased computational or manipulative skills are no longer favored. It is the standards that stimulated much of the research into what practices produce students who can demonstrate, communicate, and prove their reasoning, solve problems, and evaluate logical and mathematical arguments. In turn, some of these effective teaching practices have been incorporated into the Illinois Content Area Standards, prepared by the Illinois State board of Education. For example, teachers are required to create environments where students can work collaboratively in various groupings (Standard 1D), to use and build upon students' different thinking strategies (Standard 1E), to use problems and models and many strategies for solving them (Standards 2A and 2B), to use mathematical reasoning (Standard 3B), and to connect the different branches of mathematics to each other and to other disciplines (Standard 4D and 4E). As more and more math teachers use these effective teaching practices, the whole discipline will change and the characterizations of math as boring, repetitive, and useless should diminish.

Conclusion

The higher-order thinking skills required by the standards include reasoning and proof, solving complex problems, making connections that enable one to see mathematics as an integrated whole, and communicating clearly and effectively about one's own and others' mathematical thinking. A problem-oriented curriculum that focuses on ideas, not skills, is most effective for developing such thinking. In order to encourage students to develop mathematical mental habits and to teach the necessary skills, teachers should let students derive concepts, use cooperative group work, provide for frequent mathematical communication, use multiple representations and strategies, and use real-world applications.

I plan to include all of these aspects in my classroom. I personally hated being presented with a theorem or procedure without any background, or with the disclaimer that its origin or proof was "beyond the scope" of the class and it should just be accepted. I want to make sure my students understand the derivation of ideas, or at least have an intuitive framework into which they can fit a concept. I especially believe in the power of manipulatives, diagrams, and graphing technology at the secondary level, and I want to use these representations as often as possible to augment algebraic representations. I also feel that the ability to communicate about math is a foundational skill and I want to have frequent whole-class and group discussions, as well as individual and group writing exercises as a part of my class. I am convinced that this communication will improve my students' math performance and improve their ability to reason and communicate in their other classes and real life. Finally, I will endeavor to use collaborative group work whenever possible because it combines all of these aspects (communication, multiple representations and strategies, deriving concepts) and because mathematics is fundamentally a collaborative effort – we progress by building on each other's achievements. By incorporating these practices, I hope to become an effective mathematics teacher.

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